1. How many ways can the letters in the word "HAPPY" be arranged in a circle? (A rotation of an arrangement is not considered a new arrangement.)

   (A) 5 (B) 12 (C) 24 (D) 60 (E) 120

2. In her latest game, Mary bowled 199 and this raised her average from 177 to 178. To raise her average to 179 with the next game, she must bowl

   (A) 179 (B) 180 (C) 199 (D) 200 (E) 201

3. Let $f(n)$ be a positive integer for all positive integers n, such that $f(f(n)) = f(n) + n$ for all positive integers n. If $f(1) = 2$, then what is $f(8)$?

   (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

4. A positive integer is called *ascending* if each digit in the number (except the first) is greater than the digit on its left. For example, 2478 is an ascending number. The number of ascending numbers between 4000 and 5000 is

   (A) 6 (B) 8 (C) 9 (D) 10 (E) 15

5. If n is a positive integer, then an integer that is always divisible by 3 is

   (A) $(n+1)(n+4)$ (B) $n(n+2)(n+6)$
   (C) $n(n+2)(n+4)$ (D) $n(n+3)(n-3)$
   (E) $(n+2)(n+3)(n+5)$

6. A point P is chosen at random inside equilateral triangle ABC. What is the probability that $\angle APB$ is obtuse?

   (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{6}$ (D) $\frac{\pi\sqrt{3}}{6}$ (E) $\frac{1}{2} + \frac{\pi\sqrt{3}}{18}$

7. A club with 100 members has a telephone call system. Certain members of the club, including the president of the club, maintain a list of up to three people. To make an announcement, the president calls each person on his list. In turn, every member who has a list and receives a call forwards the message to everyone on his or her list. If all 100 members can be contacted under this system, then the maximum number of members who do not have to have a list is

   (A) 33 (B) 34 (C) 66 (D) 67 (E) 75

8. Let A, B, C, and D be four consecutive vertices of a regular octagon of side length 1. Let M be the midpoint of $\overline{AD}$. What is $MB + MC$?

   (A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}}{6}$ (D) $\frac{3}{4}$ (E) $\sqrt{2}$

9. If $x^2 + 4x + 6$ is a factor of $x^4 + rx^2 + s$, then $r + s$ is

   (A) 10 (B) 32 (C) 40 (D) 42 (E) 52

10. Pentagon $ABCDE$ with all sides having length 2 satisfies $\angle A = 90°$ and $\angle C = \angle E$. Find the area of the pentagon.

    (A) 4 (B) 6 (C) $4 + \sqrt{3}$ (D) $4 - \sqrt{3}$ (E) $4\sqrt{3}$

11. The sum $1^4 + 2^4 + \cdots + n^4$ is given by the expression

$$\frac{6n^5 + an^4 + bn^3 - n}{30},$$

for some constants a and b. The value of $a - b$ is

(A) -25 (B) -15 (C) -5 (D) 5 (E) 25

12. The digits 1, 2, 3, 4, 5, and 6 are each used once to form a six digit number $abcdef$, such that the three digit number abc is divisible by 4, bcd is divisible by 5, cde is divisible by 3, and def is divisible by 11. The digit a is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

13. The expression $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$ is equal to

(A) $\sqrt[3]{9-4\sqrt{5}}$ (B) 1 (C) $\frac{3}{2}$ (D) $\sqrt[3]{4}$ (E) $2\sqrt[3]{2}$

14. The same five numbers appear in each of the following chains of inequalities. Which chain has the correct ordering of the five numbers?

(A) $2^{3^2} < 2^{3^3} < 3^{2^2} < 3^{2^3} < 3^{3^2}$ (B) $3^{2^2} < 3^{2^3} < 2^{3^2} < 2^{3^3} < 3^{3^2}$
(C) $3^{2^2} < 3^{2^3} < 2^{3^2} < 3^{3^2} < 2^{3^3}$ (D) $3^{2^2} < 2^{3^2} < 3^{2^3} < 2^{3^3} < 3^{3^2}$
(E) $3^{2^2} < 2^{3^2} < 3^{2^3} < 3^{3^2} < 2^{3^3}$

15. Let the numbers $a_1, a_2, \ldots, a_{20}$ be a permutation of the numbers 1, 2, …, 20. How many such permutations have the property that if m divides n, then a_m divides a_n?

(A) 1 (B) 2 (C) 6 (D) 24 (E) 48

1. How many pairs of integers (x, y) satisfy $|x+y| \leq 4$ and $|x-y| \leq 4$?

2. Steve counted the number of digits among the page numbers of his history textbook, and found that there were a total of 726 digits. What is the highest page number?

3. Two rectangular blocks have the same surface area and the same volume. If the first block has dimensions $4 \times 10 \times x$ and the second block has dimensions $5 \times 6 \times y$, for some positive real numbers x and y, then find the common volume.

4. The positive integer 7654321 is *not* divisible by 11. Find the greatest positive integer that can be formed by changing the order of the seven digits that is divisible by 11.

5. Quadrilateral $ABCD$ has right angles at B and D. If $AB = AD = 20$ and $BC = CD = 15$, then find the radius of the circle inscribed in quadrilateral $ABCD$.

6. Two fair dice with faces labeled 1 through 6 are rolled and the sum of the numbers showing is recorded. This process is then repeated. What is the probability that the first sum obtained is less than the second sum?

7. One of the roots of
$$\frac{x^2+1}{x} + \frac{x}{x^2+1} = \frac{29}{10}$$
is $(1+\sqrt{k})/5$, where k is a negative integer. Find k.

8. Circle ω_1 having center O_1 and radius 21 and circle ω_2 having center O_2 and radius 6 do not intersect. Points A and C are on ω_1, and B and D on ω_2, such that lines AB and CD are both tangent to each circle. Segment O_1O_2 intersects segment CD but not AB, and CD is extended past D to meet AB at Q. If $QO_2 = 8$, then what is QO_1?